Math 2E Quiz 3 Morning - April 14th
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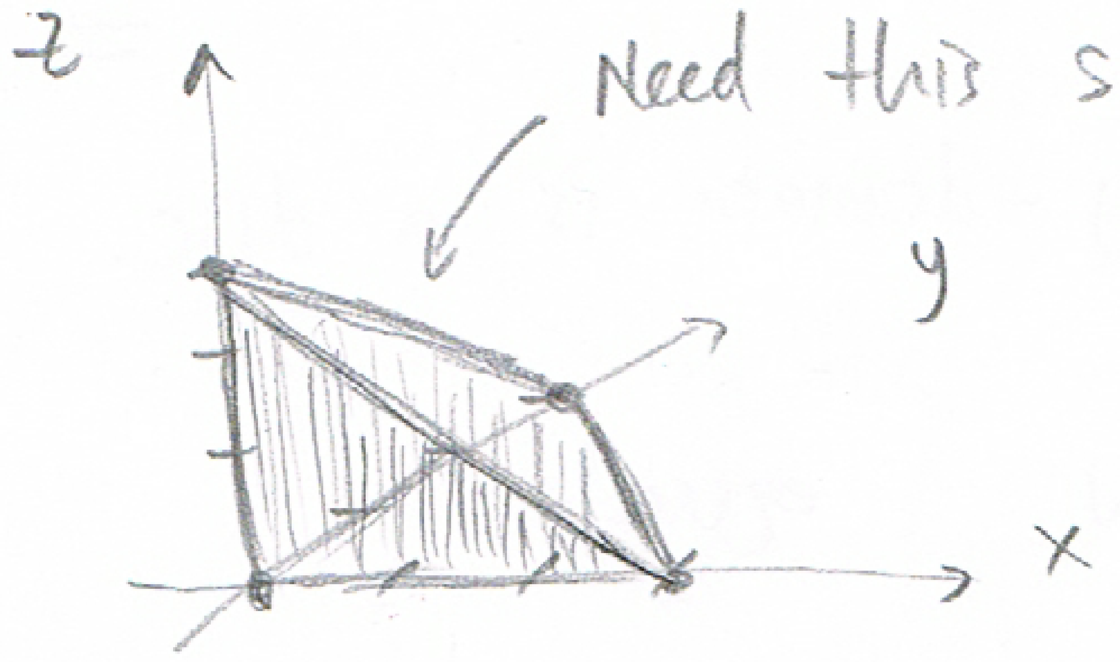
Show all of your work, and simplify all your answers. *There is a question on the back side.

1. Set up the integral that would compute $\iiint\limits_E y^2 dV$ where E is the solid tetrahedron with vertices at (0,0,0), (3,0,0), (0,3,0), (0,0,3). You do not need to evaluate this integral for full credit.
Bonus [1pt] Compute this integral.



Need this surface's eqn → <u>plane</u> w/ the pts

$(0,3,0), (3,0,0), (0,0,3)$

let $\vec{v_1} = \langle 3,-3,0\rangle$

$\vec{v_2} = \langle 3, 0, -3\rangle$

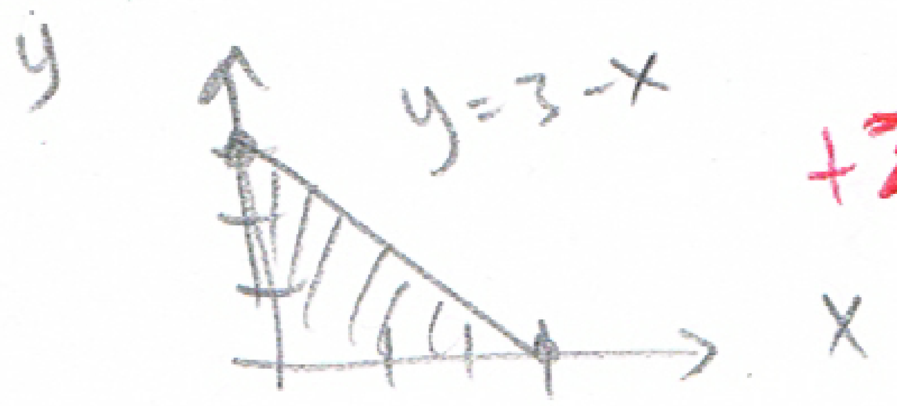
Then $\vec{n} = \vec{v_1}\times\vec{v_2} = \det\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & 0 \\ 3 & 0 & -3\end{vmatrix} = 9\hat{i} + 9\hat{j} + 9\hat{k}$ +3

$= \langle 9,9,9\rangle$ is the plane's normal

∴ Eqn of plane is (used pt (0,3,0)) $9x + 9(y-3) + 9z = 0$ +2

↳ solve z, $\boxed{z = 3-x-y}$ ~ <u>top fn</u>.

Note: xy shadow is



+2

$$\Rightarrow \boxed{\iiint\limits_E y^2 dV = \int_0^3\int_0^{3-x}\int_0^{3-x-y} y^2\, dz\,dy\,dx}$$ +3

<u>compute:</u> $= \int_0^3\int_0^{3-x}\left(y^2(3-x) - y^3\right)dy\,dx = \int_0^3 \frac{(3-x)y^3}{3} - \frac{y^4}{4}\Big|_0^{3-x} dx$

$= \int_0^3\left(\frac{(3-x)^4}{3} - \frac{(3-x)^4}{4}\right)dx = \int_0^3 \frac{(3-x)^4}{12}dx$

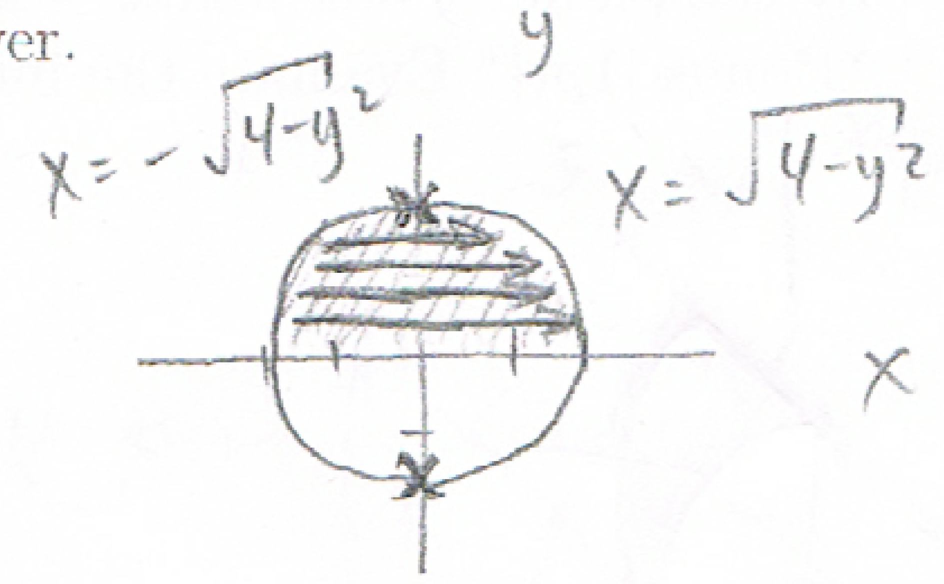
$= \ominus \frac{(3-x)^5}{60}\Big|_0^3 = \boxed{+\frac{3^5}{60}}$ (or, $\frac{81}{20}$) (+1 Bonus)

I used "*" to denote the start and end of each $x = \pm\sqrt{4-y^2}$

2. Evaluate the following integral by changing to cylindrical coordinates:

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, dz \, dx \, dy.$$

It may help, as usual, to draw the xy domain that is integrated over.



We see: $-\sqrt{4-y^2} \le x \le \sqrt{4-y^2}$, $0 \le y \le 2$ $\Rightarrow$

$\Rightarrow$ In polar, $r \in (0,2)$ and $\theta \in (0, \pi)$ then.

Also, $x, y \to r\cos\theta, r\sin\theta$, and $dx\,dy \to r\,dr\,d\theta$

$$\Rightarrow \int_0^{\pi} \int_0^2 \int_r^2 z \cdot r\cos\theta \cdot r \, dz \, dr \, d\theta$$

+6

You could iterate up to the θ-integral, but observe there is only a $\cos\theta$ - function of θ, and bounds are θ-independent,

$$= \left(\int_0^{\pi} \cos\theta \, d\theta\right)\left(\int_{r=0}^{r=2} \int_{z=r}^{z=2} z r^2 \, dz \, dr\right)$$

$$= \sin\theta \Big|_0^{\pi} \left(\int_0^2 \int_r^2 z r^2 \, dz \, dr\right)$$

$$= \boxed{0}$$

+4